Currently, the most common are disk driven vacuum devices, which largely eliminate the disadvantages mentioned above (sowing machine 3P3025AH Great Plains, Quivogne Prosem P, Kuhn Planter 3, Maestra Maschio). They have relatively small dimensions, which allows them to be placed directly above the coulter without the use of seed lines, which most significantly affect the redistribution of seed intervals. In addition, they have more efficient devices for removing unnecessary seeds from the sucking holes of the seed disc and, as a rule, provide a more even output seed flow.

Among the disadvantages inherent in the most widespread vacuum seeding machines, the researchers noted an increase in uneven seed seed when increasing the number of seeding speed of the seed disk.

The advantages of sowing machines with excessive air pressure compared with vacuum are the additional important air flow functions that improve the work of the machine, namely: removal of unnecessary seeds from the cells of seeding elements and pneumotransport of seeds from the seed disk to the bottom of the furrow with a certain redistribution of uniformity (sowing machine Tempo T of the company Väderstad, Aeromat A for Becker, Massey Ferguson for MF 555).

The disadvantage inherent in all seed presses at the same time is the need to seal their seed bins to reduce air losses and improve the supply of seeds from the bunker to the working chamber, especially the Massey Ferguson MF 555 sowing machine.

structural analysis, seed drills, precise hanging, pneumatic machine seeding, advantages, disadvantages

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Oleksiy Lobok, Assos. Prof., Phd Phys. & Math. Sci., **Boris Goncharenko,** Prof., DSc.
National University of Food Technologies, Kyiv, Ukraine
Larisa Vihrova, Prof., PhD tech. sci.
Central Ukrainian National Technical Univercity, Kropyvnytskyi, Ukraine
e-mail: vihrovalg@ukr.net
Marina Sych, PhD tech. sci.
National University of Bioresources and Nature Management of Ukraine, Kyiv, Ukraine
e-mail: sm.nuft@gmail.com

The Problem of Selection of the Optimal Strategy of Minimax Control by Objects in Agricultural Production with Distributed Parameters

The problem of minimax control synthesis for objects in agricultural production that are described by a two-dimensional heat conduction equation of parabolic type is solved. It is assumed that the control object functions under uncertainty conditions, and the perturbations acting on the object belong to some given hyperelipsoid. The problem of constructing a regulator in the state of an object for cases of point and mobile limit control is considered in accordance with the integral-quadratic quality criterion. With the help of numerical optimization methods, the problem of determining the optimal location of concentrated regulators at the boundary of a rectangular region and the problem of finding the optimal law of motion of a mobile limit regulator is solved. The problem is posed and solved in the minimax formulation when there is an optimal control on the state of the object functioning under uncertainty conditions so that the regulator minimizes the maximum control error from a set of possible values, taking into account the most unfavorable perturbations that can act on the object or system. In this case, the perturbations of the object belong to a given limited region. The results of computational experiments illustrating the effectiveness of the constructed limiting concentrated and moving regulators are presented. The obtained results indicate that the controls found in the work are indeed optimal and ensure minimum errors (deviations from the given state) of the functioning of the system and energy costs for the implementation of control under given conditions and in the absence of any information on external action other than the region of permissible perturbations.

In the work, for the first time, a minimax approach was used to control the objects described by the two-dimensional parabolic type thermal conductivity equation; the theoretical positions of synthesis of minimax regulators for cases of lumped boundary (point) and moving regulators are considered; algorithmic software is developed that allows to simulate the dynamics of the constructed minimax-regulators and to investigate the corresponding transients.

minimax control, regulators, distributed parameter systems, optimization, gradient projection method, point and mobile limit controls

О.П. Лобок, доц., канд. физ.-мат. наук **Б.М. Гончаренко**, проф., д-р техн. наук
Национальный университет пищевых технологий, Киев, Украина
Л.Г. Вихрова, проф., канд. техн. наук
Центральноукраинский национальный технический университет, Кропивницкий, Украина
М.А. Сыч, канд. техн. наук
Национальный университет биоресурсов і природоиспользования Украины, Киев, Украина

Задача выбора оптимальной стратегии минимаксного управления объектами в сельскохозяйственном производстве с распределенными параметрами

В работе решается задача синтеза минимаксного управления для объектов в сельскохозяйственном производстве, которые описываются двумерным уравнением теплопроводности параболического типа. Предполагается, что объект управления функционирует в условиях неопределенности, причем возмущения, действующие на объект, принадлежат некоторому заданному гиперэлипсоиду. Рассматривается задача построения регулятора по состоянию объекта для случаев точечного и подвижного предельного управления в соответствии с интегрально-квадратичным критерием качества. С помощью числовых оптимизационных методов решена задача определения оптимального расположения сосредоточенных регуляторов на границе прямоугольной области и задача поиска оптимального закона перемещения подвижного предельного регулятора. Задача ставится и решается в минимаксной постановке, когда находится оптимальное регулирование по состоянию объекта, функционирующего в условиях неопределенности так, что регулятор обеспечивает минимизацию максимальной погрешности регулирования из множества возможных значений с учетом наиболее неблагоприятных возмущений, которые могут действовать на объект или систему. При этом возмущения объекта относятся к заданной ограниченной области. Приводятся результаты вычислительных экспериментов, иллюстрирующие эффективность построенных предельных сосредоточенных и подвижных регуляторов. Полученные результаты свидетельствуют о том, что найденные в работе управления действительно являются оптимальными и обеспечивают минимум погрешности (отклонения от заданного состояния) функционирования системы и энергетических затрат на осуществление управления при заданных условиях и при отсутствии какой-либо информации о внешнем воздействии, кроме области допустимых возмущений.

минимаксное управление, регуляторы, системы с распределенными параметрами, оптимизация, метод проекции градиента, точечное и подвижное предельные управления

Introduction. In connection with the widespread adoption of new advanced technologies related to the use of electronic, ion, laser and other radiation, in recent years intensive study of the possibilities of optimal control of distributed source systems by changing the location of point sources of radiation and the laws of motion of moving.

Statement of the problem and analysis of recent researches. The determination of the problems of point and motion control, some methods of their solution are given in the works [1,2,3]. One of the most important and complex tasks is the choice of an optimal point and move control strategy for systems that operate under uncertainty. It is this problem that is devoted to this article, which solves the problem of choosing the optimal location of point regulators and finding the optimal law of motion (moving) of a moving source at the boundary of a rectangular region for the process of heat transfer occurring under incomplete information. The theory of control moves towards the complexity of the phenomena studied, processes and the reduction of information about the control system, the object, its features, properties, characteristics, operating conditions, external influences. Taking into account all the above-mentioned, the chosen direction of research is perspective and has a high level of relevance.

The purpose of the article is the practical application of the theory of synthesis of minimax regulators to the problems of controlling the choice of optimal arrangement of point regulators and the search for an optimal motion law in conditions of uncertainty of external perturbations acting on them in the domain of their admissibility.

Presenting main material. Let the process of heat transfer in a homogeneous thin rectangular plate be described by a function $\varphi(x,t)$, which is in the area $Q_T = \Omega \times (0,T)$, where $\Omega = \{(x_1,x_2):\ 0 < x_1 < l_1,\ 0 < x_2 < l_2\}$, $l_1, l_2 > 0$, $T < \infty$, satisfies the equation

$$\frac{\partial \varphi(x,t)}{\partial t} = a\Delta_x \varphi(x,t) + f_1(x,t), \quad (x,t) \in Q_T, \tag{1}$$

but on the border Q_T − additional conditions

$$\varphi(x,0) = f_0(x),\ \ x \in \Omega; \quad \varphi(x,t) = \sum_{i=1}^{N} \delta(x - v_i(t)) u_i(t),\ \ (x,t) \in \Gamma \times (0,T). \tag{2}$$

Here $\Delta_x = \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial x_2^2}$ − two-dimensional Laplace operator;

$a > 0$ − coefficient of temperature conductivity;

Γ − border of rectangular area Ω;

$\delta(x - y)$ − Dirac's delta function;

$t \to v_i(t) \in \Gamma$ − dimensional functions that determine the motion of boundary sources;

$u_i(t) \in L_2(0,T)$ − control functions;

$f_0(x) \in L_2(\Omega)$, $f_1(x,t) \in L_2(Q_T)$ − unknown functions belonging to the area

$$S_t = \left\{ \left(f_0, f_1 \right):\ G\left(f_0; f_1(\tau),\ 0 < \tau < t \right) \le 1 \right\}, \qquad t \in (0,T], \tag{3}$$

where

$$G\left(f_0; f_1(\tau),\ 0 < \tau < t \right) = F_0 \int_{\Omega} f_0^2(x)dx + F_1 \int_0^t \int_{\Omega} f_1^2(x,\tau)dxd\tau,$$

and F_0, F_1 − positive constant values reflecting the contribution of noise f_0 and $f_1(t)$ in the final perturbation, acting on the system (1), (2).

Under the solution of the boundary value problem (1), (2) we will understand such a function $\varphi(x,t) \in L_2(Q_T)$, which satisfies the following integral identity

$$-\int_0^T \int_{\Omega} \varphi(x,t) \left(\frac{\partial \eta(x,t)}{\partial t} + a\Delta_x \eta(x,t) \right) dxdt = \int_{\Omega} f_0(x)\eta(x,0)dx + \int_0^T \int_{\Omega} f_1(x,t)\eta(x,t)dxdt -$$

$$-a\sum_{i=1}^{N} \int_0^T u_i(t) \left. \frac{\partial \eta(x,t)}{\partial n} \right|_{x=v_i(t)} dt \qquad \forall \eta(x,t) \in \Phi, \tag{4}$$

where $\partial/\partial n$ − derivative of the external normal $\vec{n}$ to the border Γ of the area Ω,

$\Phi = \left\{ \eta(x,t):\ \eta(x,t) \in H^{3,1}(Q_T),\ \eta(x,T) = 0,\ x \in \Omega;\ \eta(x,t) = 0,\ (x,t) \in \Gamma \times (0,T) \right\}$,

$H^{3,1}(Q_T)$ − Sobolovsky space [4].

It can be shown [5] that the solution of equation (4) with given controls $u_i(t) \in L_2(0,T)$ exists and is unique in space $L_2(Q_T)$.The task of choosing the optimal strategy for minimax control [6, 8] will be to find vector functions $v^*(t) = \left[v_1^*(t), v_2^*(t), ..., v_N^*(t) \right]^T$ and $u^*(t) = \left[u_1^*(t), u_2^*(t), ..., u_N^*(t) \right]^T$ under conditions

$$I(u^*, v^*) = \inf_v \inf_u I(u,v), \tag{5}$$

where

$$I(u,v) = \sup_{S_T} \left[\int_\Omega S(x)\varphi(x,T)dx \right]^2 + \int_0^T \sum_{i=1}^N d_i \sup_{S_t} u_i^2(t)dt, \tag{6}$$

$S(x) \in L_2(\Omega)$, $d_i = const > 0$, $i = 1,2,...,N$, with a given control structure $u_i(t)$ in the form of a linear feedback

$$u_i(t) = \int_\Omega R_i(x,t)\varphi(x,t)dx. \tag{7}$$

The solution of the formulated problem will be carried out in two stages: first we solve the problem of determining the optimal control $u^*(t)$ under condition

$$I(u^*, v) = \inf_u I(u,v) \tag{8}$$

for a fixed vector-function $v(t)$, and then find it $v^*(t)$, at which

$$I(u^*, v^*) = \inf_v I(u^*, v). \tag{9}$$

According to the results of [7,8], the following theorem is proved: optimal control $u^*(t)$ of the optimization problem (1), (2), (6), (7) satisfying the necessary optimality conditions has the form

$$u_i^*(t) = \int_\Omega R_i^*(x,t)\varphi(x,t)dx, \quad R_i^*(x,t) = a d_i^{-1}\alpha^{-1}(t) g(x,t) h(v_i(t),t), \tag{10}$$

where

$$\alpha(t) = 1 + a^2 \sum_{k=1}^N d_k^{-1} \int_t^T h^2(v_k(\tau),\tau)d\tau,$$

$$\begin{bmatrix} g(x,t) \\ h(x,t) \end{bmatrix} = \sum_{i=1}^\infty s_i e^{\lambda_i(t-T)} \begin{bmatrix} \omega_i(x) \\ r_i(x) \end{bmatrix}, \quad s_i = \int_\Omega S(x)\omega_i(x)dx. \tag{11}$$

In the ratio (11) $i = (i_1, i_2)$ − multiindex

$$r_i(x) = r_{i_1,i_2}(x_1,x_2) = \frac{2\pi}{\sqrt{l_1 l_2}} \begin{cases} (-1)^{i_1}\dfrac{i_1}{l_1}\sin\dfrac{\pi i_2 x_2}{l_2}, & x_1 = l_1, \\ -\dfrac{i_1}{l_1}\sin\dfrac{\pi i_2 x_2}{l_2}, & x_1 = 0, \\ (-1)^{i_2}\dfrac{i_2}{l_2}\sin\dfrac{\pi i_1 x_1}{l_1}, & x_2 = l_2, \\ -\dfrac{i_2}{l_2}\sin\dfrac{\pi i_1 x_1}{l_1}, & x_2 = 0, \end{cases} \qquad x = (x_1, x_2) \in \Gamma,$$

where λ_i, $\omega_i(x)-$ eigenvalues and corresponding orthonormalities in space $L_2(\Omega)$ own functions of the boundary value problem (1), (2) having the form

$$\lambda_i = \lambda_{i_1,i_2} = a\pi^2\left[(i_1/l_1)^2 + (i_2/l_2)^2\right],$$

$$\omega_i(x) = \omega_{i_1,i_2}(x_1,x_2) = \frac{2}{\sqrt{l_1 l_2}}\sin\frac{\pi i_1 x_1}{l_1}\sin\frac{\pi i_2 x_2}{l_2}.$$

The value of the functional (6) for optimal control (10) is determined by the formula

$$I(u^*, v) = \frac{W(0)}{F_0\alpha(0)} + \frac{1}{F_1}\int_0^T \frac{W(t)}{\alpha(t)}dt, \tag{12}$$

where

$$W(t) = \sum_{i=1}^{\infty} s_i e^{2\lambda_i(t-T)}. \tag{13}$$

Let us now turn to the solution of the optimization problem (9), (12). Let's consider first a simpler case when $v_i(t) \equiv z_i \in \Gamma$, $i = 1, 2, ..., N$, that is, we solve the problem of optimal location of point boundary controls (10). Let's introduce the designation $z = [z_1, z_2, ..., z_N]^T$, $J(z) = I(u^*, z) \equiv I(u^*, v)$. Then the task under consideration will be to find the vector $z = [z_1^*, z_2^*, ..., z_N^*]^T$, at which

$$J(z^*) = \inf_{z\in\Omega_z} J(z) \tag{14}$$

where $\Omega_z = \left\{z:\ z = [z_1, z_2, ..., z_N]^T,\ z_i = (z_{1i}, z_{2i}) \in \Gamma,\ i = 1, 2, ..., N;\ z_i \neq z_j,\ i \neq j\right\}$.

Given that function $J(z)$ is a continuously differentiated function of its arguments, to solve the optimization problem (14) we use the gradient projection method [9]

$$z^{k+1} = \mathrm{Pr}_{\Omega_z}\left[z^k - \rho_k \nabla_z J(z^k)\right], \quad k = 0, 1, 2, ..., \tag{15}$$

where $\mathrm{Pr}_{\Omega_z}[z] = [y_1, y_2, ..., y_N]^T$, $y_i = \mathrm{Pr}_\Gamma[z_i]$ − projection of point z_i on the border Γ of a rectangular area Ω;

$z^k = \left[z_1^k, z_2^k, ..., z_N^k\right]^T$ − approximate solution obtained on *k*-th iteration;

z^0 − initial approximation;

ρ_k − step of descent, which is chosen from the condition of the monotonous decline of the function of purpose $J(z)$ [9]; gradient $\nabla_z J(z)$ is determined by the formula

$$\nabla_z J(z) = \left[\nabla_{z_1} J(z), \nabla_{z_2} J(z), ..., \nabla_{z_N} J(z) \right]^T ,$$

$$z = \left[z_1, z_2, ..., z_N \right]^T , \quad z_i = \left(z_{1i}, z_{2i} \right) \in \Gamma, \; i = 1,2,...,N ,$$

$$\nabla_{z_n} J(z) = -2a^2 \left[\frac{W(0)\theta_n(0)}{F_0 \alpha^2(0)} + \frac{1}{F_1} \int_0^T \frac{W(t)\theta_n(t)}{\alpha^2(t)} dt \right],$$

$$\theta_n(t) = d_n^{-1} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{1 - e^{(\lambda_i + \lambda_j)(t-T)}}{\lambda_i + \lambda_j} s_i s_j r_i(z_n) P_j(z_n) ,$$

where $P_j(z_n) = \left[P_j^1(z_n), P_j^2(z_n) \right]$,

$$P_j^k(z_n) = P_{j_1,j_2}^k \left(z_{1n}, z_{2n} \right) = \frac{2\pi^2 j_1 j_2}{(l_1 l_2)^{3/2}} \cos \frac{\pi j_k z_{kn}}{l_k} \begin{cases} 0, & z_{kn} = 0, \; z_{kn} = l_k , \\ -1, & z_{3-k,n} = 0, \\ (-1)^{j_{3-k}} , & z_{3-k,n} = l_{3-k} , \end{cases} \quad k = 1,2 .$$

The condition of the stop was taken in the form $\left| J(z^{k+1}) - J(z^k) \right| < \varepsilon$, where $\varepsilon > 0$ − the accuracy of the solution is given.

This algorithm was programmed in the algorithmic language Fortran 90 with the following initial data: $l_1 = 2.0$, $l_2 = 1.0$, $T = 2.0$, $F_0 = 0.25$, $F_1 = 2.0$, $d_i = 1.0$, $i = 1,2,...,N$, $S(x) = 1.0$, $\varepsilon = 0.001$, $\rho_0 = 0.8$, number of regulators $N = 5$, for the value of the coefficient of thermal conductivity a 0.4 was taken, which corresponds to the coefficient of thermal conductivity of the copper plate. The dimension of all quantities is given in the system [meter, time, deg. C^o, kcal.]. The infinite series (11), (13) were broken off by the finite sum of the three first members. For numerical simulation of optimal controls $u_i^*(t)$ it was assumed that perturbation $f_0(x)$ and $f_1(x,t)$ is equal

$$f_0(x_1, x_2) = 2 \sin \frac{\pi x_1}{l_1} \sin \frac{\pi x_2}{l_2} , \qquad f_1(x_1, x_2, t) = t \sin \frac{\pi x_1}{l_1} \cos \frac{\pi x_2}{l_2} .$$

We note that these perturbations are permissible, because $G\left(f_0, f_1(\tau), 0 < \tau < t \right) = F_0 l_1 l_2 + \frac{1}{12} F_1 l_1 l_2 t^3 = 0.5 + \frac{1}{3} t^3 < 1, \quad \forall \; t \in (0, 0.2)$ and as a result, $f_0(x)$ and $f_1(x,t)$ belong to the area (3).

Table 1 gives the initial location $z^0 = \left[z_1^0, z_2^0, ..., z_N^0 \right]^T$ point boundary regulators. Function value $J(z)$ at such an arrangement of controls equals $J(z^0) = 0.975632$. Optimal arrangement of regulators $z^* = \left[z_1^*, z_2^*, ..., z_N^* \right]^T$, obtained by the algorithm (15), is given in the table 2 and $J(z^*) = 0.571874$.

Table 1.

k	z_{1k}^0	z_{2k}^0
1	2.0	0.0
2	1.0	0.0
3	0.0	0.0
4	0.0	0.5
5	0.0	1.0

Table 2.

k	z_{1k}^*	z_{2k}^*
1	1.349	0.0
2	1.0	0.0
3	0.651	0.0
4	0.0	0.5
5	0.651	1.0

Figure 1 shows graphs of optimal point controls (10) that are optimally located on the boundary Γ of the area Ω в точках $z_i^* \in \Gamma,\ i = 1, 2, \ldots, N$.

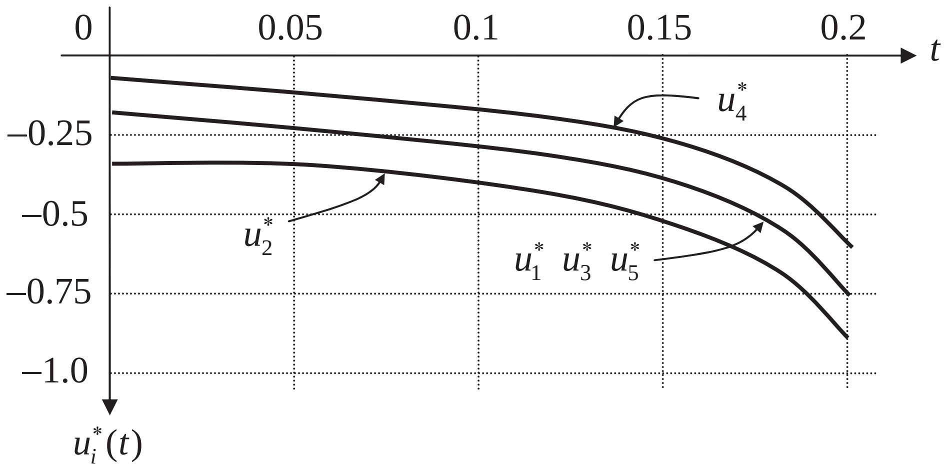

Figure 1 – Schedule of optimal boundary point controls

We now turn to the optimization problem (9), (12). Let's lick for simplicity one $(N = 1)$ a moving source and let the perturbation $f_1(x,t)$ in the right side of equation (1) will be absent. Let's denote $u(t) = u_1(t)$, $v(t) = v_1(t)$, $d = d_1$, $J(v) = I(u^*, v)$. Then the task of minimizing the functional

$$J(v) = W(0)(F_0\alpha(0))^{-1} \tag{16}$$

is equivalent to the next optimization problem

$$L(v) = \int_0^T h(v(\tau),\tau)d\tau \to \sup_{\substack{t \to v(t) \in \Gamma \\ t \in (0,T)}},$$

where $\alpha(t)$, $h(x,t)$, $W(t)$ − functions determined by the formulas (11), (13).

To solve the last problem, the projection method of the gradient of the species was also used

$$v^{k+1}(t) = \mathrm{Pr}_\Gamma\left[v^k(t) + \rho_k \delta[L(v^k);t]\right], \quad t \in (0,T), \qquad k = 0,1,2,..., \tag{17}$$

where $v^0(t)$ − initial approximation;

$v^k(t)$ − Approximate solution obtained at k-th step;

ρ_k − step of descent to the minimum point;

$\delta[L(v);t]$ − graceful Frechet functional $L(v)$ which is calculated by the formula

$$\delta[L(v);t] = 2h(v(t),t)\rho(v(t),t)\,, \qquad \rho(x,t) = \sum_{i=1}^{\infty} s_i e^{\lambda_i (t-T)} P_i(x)\,.$$

The algorithm stops when is fulfilled the condition $\left|L(v^{k+1}) - L(v^k)\right| < \varepsilon$, where $\varepsilon > 0$ − the accuracy of the solution is given.

Numerical implementation of the algorithm (17) was carried out with the above earlier data. Below are the results of computational calculations. In table 3 the initial law of motion is given $v^0(t) = \left(v_1^0(t), v_2^0(t)\right)$ for a moving boundary source. Optimal motion law $v^*(t) = \left(v_1^*(t), v_2^*(t)\right)$ of the moving controller (10), obtained by the algorithm (17), is given in table 4.

Table 3.

t	$v_1^0(t)$	$v_2^0(t)$
0.0	0.0	0.0
0.02	0.667	0.0
0.04	1.333	0.0
0.06	2.0	0.0
0.08	2.0	0.5
0.10	2.0	1.0
0.12	1.333	1.0
0.14	0.667	1.0
0.16	0.0	1.0
0.18	0.0	0.5

Table 4.

t	$v_1^*(t)$	$v_2^*(t)$
0.0	0.010	0.0
0.02	0.765	0.0
0.04	1.230	0.0
0.06	1.990	0.0
0.08	2.0	0.5
0.10	1.990	1.0
0.12	1.285	1.0
0.14	0.664	1.0
0.16	0.010	1.0
0.18	0.0	0.5

The value of the functional (16) thus decreased from $J(z^0) = 0.639538$ to $J(z^*) = 0.438419$. In fig. 2 shows an optimal control (10), the movement of which is carried out in the optimal trajectory, shown in the table 4. The optimal trajectory consists of four parts, each of which resembles a parabola and defines (describes) the motion of the regulator along the corresponding boundary of the rectangular area.

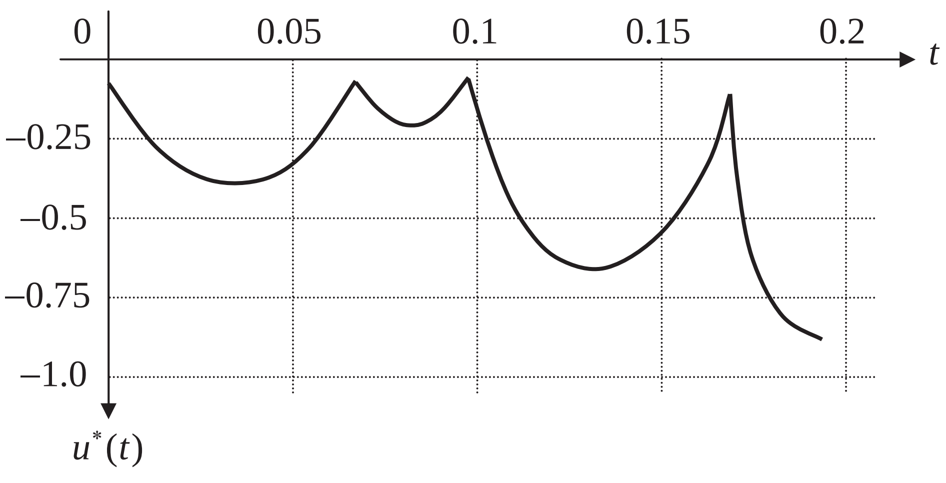

Figure 2 – Schedule of Optimum Limit Moving Control

The computational experiments also showed that the efficiency of point and moving boundary controls increases with a decrease in the coefficient of temperature conductivity, that is, with the decrease of this coefficient the value of the functional (8) after determining the optimal control strategy decreased by a larger value compared with the value of the same functional with a given initial control strategy.

Conclusions. In this paper the solution of the problem of finding the optimal placement strategy for point boundary regulators and the problem of determining the optimal trajectory of moving a regulator along the boundary of the region in which the distributed control object functions is achieved. The problem is solved and solved in a minimal-scale setting, that is, an optimal controller is found for the state of the object, which functions in conditions of uncertainty, and the perturbation of the object belongs to a given bounded domain. The results of computational experiments are presented, which illustrate the efficiency of constructed lumped boundary point and moving regulators. The obtained results indicate that the control outputs are actually optimal and provide a minimum of errors (deviations from the given state) of the system's operation and energy costs for the control of the given conditions and the absence of any information on external influences, in addition to the area of permissible perturbations. Satisfactory performance indicators are observed even in the event of disturbance beyond the boundaries of a given area.

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О.П. Лобок, доц., канд. фіз.-мат. наук**, Б.М. Гончаренко,** проф., д-р техн. наук
Національний університет харчових технологій, Київ, Україна
Л.Г. Віхрова, проф., канд. техн. наук
Центральноукраїнський національний технічний університет, Кропивницький, Україна
М.А. Сич, канд. техн. наук
Національний університет біоресурсів и природокористування України, м.Київ, Україна

Задача вибору оптимальної стратегії мінімаксного керування в сільськогосподарському виробництві об'єктами з розподіленими параметрами

В роботі розв'язується задача синтезу мінімаксного керування для об'єктів сільськогосподарського виробництва, які описуються двовимірним рівнянням теплопровідності параболічного типу. Передбачається, що об'єкт керування функціонує в умовах невизначеності, причому збурення, що діють на об'єкт, належать деякому заданому гіпереліпсоїду. Розглядається задача побудови регулятора стану об'єкта для випадків точкового і рухомого граничного керування згідно з інтегрально-квадратичним критерієм якості. За допомогою числових оптимізаційних методів розв'зана задача визначення оптимального розташування зосереджених регуляторів на кордоні прямокутної області і завдання пошуку оптимального закону переміщення рухомого граничного регулятора. Задача ставиться і розв'зується в мінімаксної постановці, коли знаходиться оптимальне регулювання станом об'єкта, який функціонує в умовах невизначеності так, що регулятор забезпечує мінімізацію максимальної похибки регулювання з безлічі можливих значень з урахуванням найбільш несприятливих збурень, які можуть діяти на об'єкт або систему. При цьому збурення об'єкта стосується до заданої обмеженої області. Наводяться результати обчислювальних експериментів, що ілюструють ефективність побудованих граничних зосереджених і рухомих регуляторів.

Отримані результати свідчать про те, що знайдені в роботі керування дійсно є оптимальними і забезпечують мінімум похибки (відхилення від заданого стану) функціонування системи і енергетичних витрат на здійснення керування при заданих умовах і при відсутності будь-якої інформації про зовнішні впливи, крім області допустимих збурень.

мінімаксне керування, регулятори, системи з розподіленими параметрами, оптимізація, метод проекції градієнта, точкове і рухоме граничне керування

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